day 39
(yesterday)

$a^m \cdot a^n = a^{m+n}$

$\frac{a^m}{a^n} = a^{m-n}$

$(a^m)^n = a^{mn}$

$a^{-m} = \frac{1}{a^m}$

$a^0 = 1$

$a^1 = a$

$a^{\frac{1}{m}} = \sqrt[m]{a}$

$(abc)^m = a^m b^m c^m$

$\left[\frac{a^{m_1} b^{m_2} c^{m_3}}{d^{m_4} e^{m_5}}\right]^n = \frac{a^{nm_1} b^{nm_2} c^{nm_3}}{d^{nm_4} e^{nm_5}}$

$\log(ab) = \log a + \log b$

$\log \frac{a}{b} = \log a - \log b$

$\log a^m = m \log a$

$e^{\ln a} = a \; , \; \ln e^a = a$

$\log_b a = x \Leftrightarrow b^x = a$

Oct 24-11:42 AM

day 40

3.8/25) $x^2+xy+y^2=7$; $(2,1)$

WITHOUT A PADDLE

$$\frac{d}{dx}(x^2)+\frac{d}{dx}(xy)+\frac{d}{dx}(y^2)=\frac{d}{dx}(7)$$

product Rule

$$2x+\left[y+x\frac{dy}{dx}\right]+2y\frac{dy}{dx}=0$$

$$x\frac{dy}{dx}+2y\frac{dy}{dx}=-2x-y$$

$$\frac{dy}{dx}(x+2y)=-2x-y$$

$$\frac{dy}{dx}=\frac{-2x-y}{x+2y}=-\frac{2x+y}{x+2y}$$

day 40

3.8/32/ $y = \sqrt[3]{x^2 - x + 1} = (x^2 - x + 1)^{1/3}$

$$y' = \frac{1}{3}(x^2 - x + 1)^{\frac{1}{3} - 1} \cdot \frac{d}{dx}(x^2 - x + 1)$$

$$= \frac{1}{3}(x^2 - x + 1)^{-\frac{2}{3}}(2x - 1)$$

$$= \frac{2x - 1}{3\sqrt[3]{(x^2 - x + 1)^2}}$$ (simplified)

day 40

3.8/59)

$$x+y^3-xy=1$$

waving hands . . . .

$$y^3-1=xy-x$$

$$\frac{(y-1)(y^2+y+1)}{y-1}=\frac{x(y-1)}{y-1}$$

$$y^2+y+1=x$$

Ⓐ Let $y=1$

then $x+1-x=1$

$1=1$

$\Rightarrow$ all values of x work

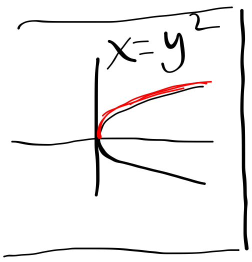
Question: WE assume $y\neq 1$ here. is that introducing a new restriction?

a) $2y\frac{dy}{dx}+\frac{dy}{dx}+0=1$

$$\frac{dy}{dx}(2y+1)=1 \Rightarrow \frac{dy}{dx}=\frac{1}{2y+1}$$

b) $y^2+y+1=x$

$$y^2+y+(1-x)=0 \Rightarrow y=\frac{-1\pm\sqrt{1-4(1)(1-x)}}{2}$$



$$y=\frac{-1\pm\sqrt{4x-3}}{2}$$

$$y=-\frac{1}{2}\pm\frac{1}{2}\sqrt{4x-3}$$

from original

$$y'=\pm\frac{1}{2}(4)\left(\frac{1}{2}\right)\frac{1}{\sqrt{4x-3}}$$

from $\frac{d}{dx}\sqrt{x}$

from $\frac{d}{dx}(4x-3)$

$$\frac{d}{dx}\left((4x-3)^{1/2}\right)$$

$$=\frac{1}{2}(4x-3)^{-\frac{1}{2}}\cdot\frac{d}{dx}(4x-3)$$

day 40

$$y^2+y+1=x$$

alternative

$$y^2+y+\left(\tfrac{1}{2}\right)^2-\left(\tfrac{1}{2}\right)^2+1=x$$

$$\left(y+\tfrac{1}{2}\right)^2+\tfrac{3}{4}=x$$

$$\left(y+\tfrac{1}{2}\right)^2=x-\tfrac{3}{4}$$

$$y+\tfrac{1}{2}=+\sqrt{x-\tfrac{3}{4}}$$

or

$$-\sqrt{x-3/4}$$

day 40

3.7

69c) The derivative of a product is NOT the product of THE derivatives,

but the derivative of a composition is A product of derivatives.

FALSE that $\frac{d}{dx}(f\cdot g) = f'g'$

A composition of 2 f^{ns} $\Rightarrow$ derivative 2 factors

$$\frac{d}{dx}\left(f(g(x))\right) = \underbrace{f'(g(x))}_{1\text{ factor}} \cdot \underbrace{g'(x)}_{2\text{ factor}}$$

A comp of 3 f^{ns} $\Rightarrow$ derivative ≥ 3 factors

$$\frac{d}{dx}\left(f(g(h(x)))\right) = f'(g(h(x))) \cdot g'(h(x)) \cdot h'(x)$$

Oct 24-11:44 AM

day 40

3.8/69) $3x^3+7y^3=10y$; $(1,1)$

find $\frac{dy}{dx}$

$$\frac{d}{dx}(3x^3)+\frac{d}{dx}(7y^3)=\frac{d}{dx}(10y)$$

$$9x^2+21y^2\frac{dy}{dx}=10\frac{dy}{dx}$$

$$21y^2\frac{dy}{dx}-10\frac{dy}{dx}=-9x^2$$

$$\frac{dy}{dx}=\left(\frac{-9x^2}{21y^2-10}\right)$$

at $(1,1)$: $\frac{dy}{dx}=\frac{-9(1)^2}{21(1)^2-10}=\frac{-9}{11}$

so slope of tangent line at $(1,1)=-\frac{9}{11}$

slope of normal line at $(1,1)=+\frac{11}{9}$

Oct 24-11:44 AM

day 40

e^y $e^{f(x)}$ $y=f(x)$

3.10) starting from

$$\sin\left(\sin^{-1}(x)\right) = x$$

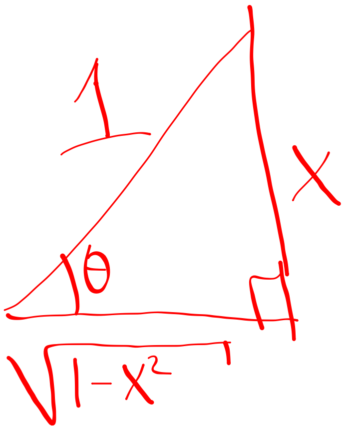
diff implicitly

$$\cos\left(\sin^{-1}(x)\right) \cdot \frac{d}{dx}\left(\sin^{-1}(x)\right) = 1$$

So $\dfrac{d}{dx}\left(\sin^{-1}(x)\right) = \dfrac{1}{\cos\left(\sin^{-1}(x)\right)}$

but what does this mean?

$\theta = \sin^{-1}(x) \doteq$ "the angle whose sine is x"



$$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\cos\left(\sin^{-1}(x)\right)}$$

$$= \frac{1}{\cos(\theta)} = \frac{1}{\sqrt{1-x^2}}$$

So

$$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$$

Oct 24-11:44 AM

day 40

HW/ 3.10/1-8

3.9/ 51-52, 55, 61-62, 69

Oct 24-11:44 AM